# An Investigation into Optimal Spending for the Government of British Columbia

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## 1 Introduction

The economic growth of a nation is a key factor that determines its overall prosperity. As such, governments are often tasked with making strategic investments to maximize the Gross Domestic Product (GDP) of their respective countries. In British Columbia (BC), the government is faced with the challenge of determining how to invest its financial resources across different sectors in order to achieve the greatest possible economic impact. This is a complex problem that can be addressed through the use of linear programming techniques. Linear programming is a powerful tool for optimizing resource allocation and decision-making. In the context of government policies and investments, it can be used to determine the optimal allocation of resources across various sectors to achieve a desired outcome.

In this paper, we will be answering the question: *How should the BC government allocate their funding to various programs and services to minimize BC's Expense to GDP ratio?* We will also be discussing the insights our model gives, some of it's pitfalls and more importantly, areas of improvement for future models.

# 2 Data

## 2.1 Sources

We will be using two data sources:

1. BC's GDP data from 2000 to 2021.[1]
2. BC's financial information from 1997 to 2021.[2], ... ,[26]

## 2.2 Cleaning and Wrangling

BC's financial information from 1997 to 2021 categorized its expenses into 7 categories: health, education, social services, natural resources and economic development, transportation, general government expenses and other. We will be using the same expense categories for our LP problem.

In order to clean and wrangle our data into a more useable dataset, we had to aggregate our GDP data into the 7 expense categories. The BC GDP data from 2000 to 2021 had GDP categorized in more categories than we needed.

Originally, the GDP data was categorized into the following, separated by a semicolon:

Agriculture, forestry, fishing and hunting; Mining, quarrying, and oil and gas extraction; Utilities; Construction; Manufacturing; Wholesale trade; Retail trade; Transportation and warehousing; Information and cultural industries; Finance and insurance; Real estate and rental and leasing; Professional, scientific and technical services; Management of companies and enterprises; Administrative and support, waste management and remediation services; Educational services; Health care and social assistance; Arts, entertainment and recreation; Accommodation and food services; Other services (except public administration); Public administration.

From there, we grouped the GDP categories under one of the expense categories. The grouping is not the most accurate because our expenses are only discretized into 7 categories, so there isn't a lot of room to fine-tune the grouping. The grouping is as follows, separated by a semicolon:

**Health:** Health care and social assistance

**Education:** Educational services

**Social Services (Excluding Social Assistance):** Accommodation and food services; Other services (except public administration); Arts, entertainment and recreation; Administrative and support, waste management and remediation services; Utilities

**Natural Resources and Economic Development:** Construction; Manufacturing; Mining, quarrying, and oil and gas extraction; Agriculture, forestry, fishing and hunting

**Transportation:** Transportation and warehousing

**General Government:** Public administration

**Other:** Information and cultural industries; Finance and insurance; Wholesale trade; Retail trade; Real estate and rental and leasing; Management of companies and enterprises; Professional, scientific and technical services

## 3 Preliminary Results

Figure 1 is our cleaned historical GDP per category data, figure 2 is our cleaned BC Government expenses per category, figure 3 and figure 4 is our cleaned revenue and real GDP data respectively. We have left out 2022 data because the real GDP dollar amount is not available. For years 2000[23] and 2002[21], the BC Government's expenses were not given as a dollar amount, it was a proportion of total expenses. Therefore, we computed a dollar amount using the proportion of expenditures for each category.

Some initial observations are that the health category has the highest expenses, but its GDP is on the lower end. The other category has the highest GDP and the historical expenditure is modest. This has to do with the other category having quite a lot of sectors grouped together. These sectors don't fully fit into the defined expense categories, but removing data is not ideal, so we grouped them together. Education also has quite a low historical GDP and high expenses but this is only how much money educational services in itself create. Investment into education creates a lot of value externally as it helps to develop more skilled workers, this external value it creates is not reflected within the education category. The natural resources and economic development category also has quite a high historical GDP and low historical investment from the government.

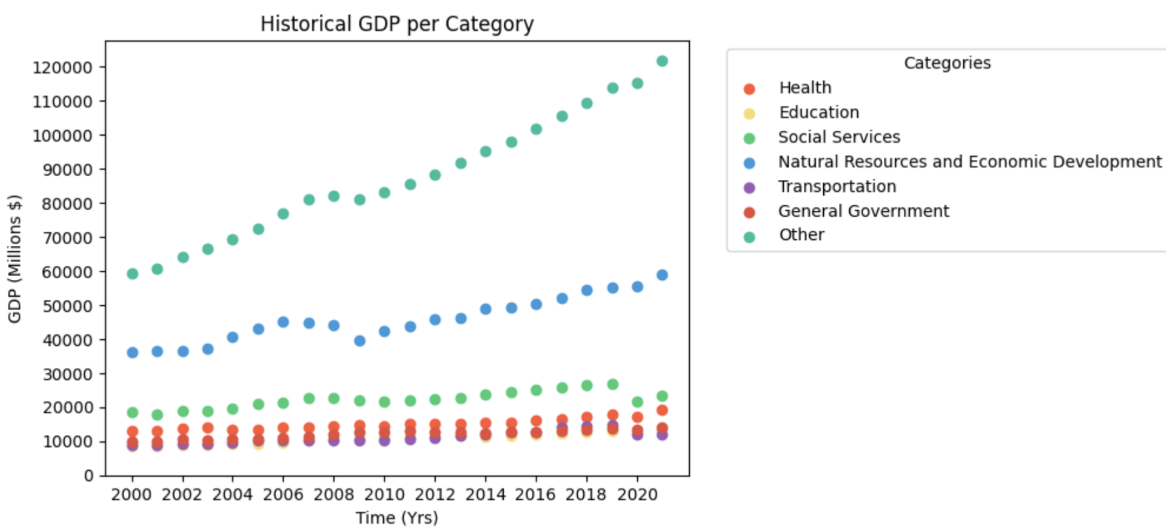



Figure 1: Historical GDP per Category

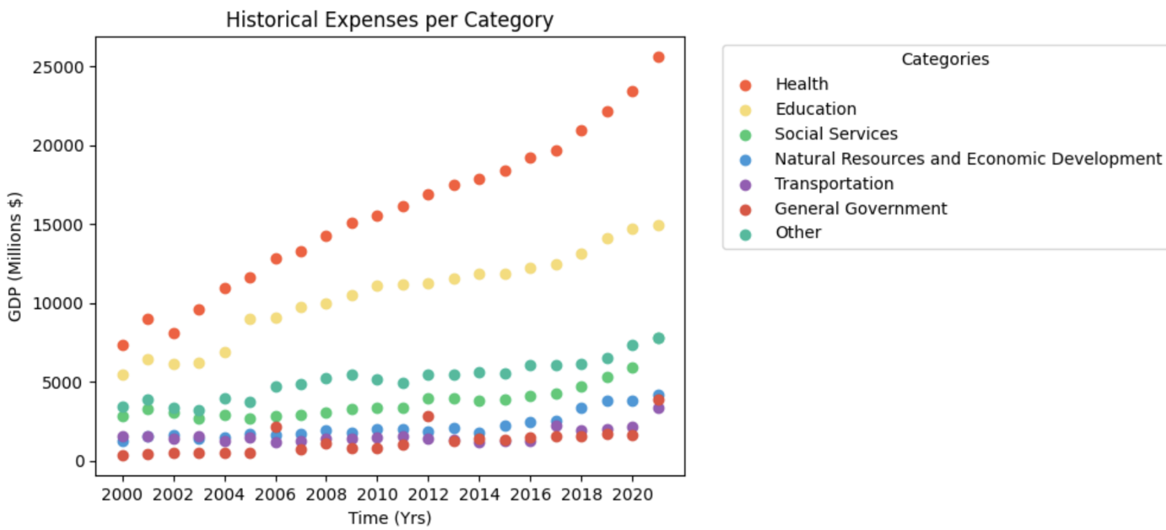



Figure 2: Historical Expenses per Category

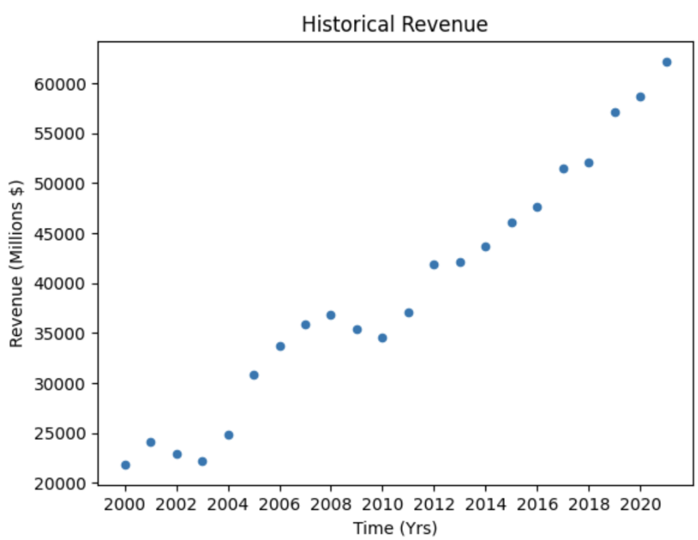



Figure 3: Historical Revenue

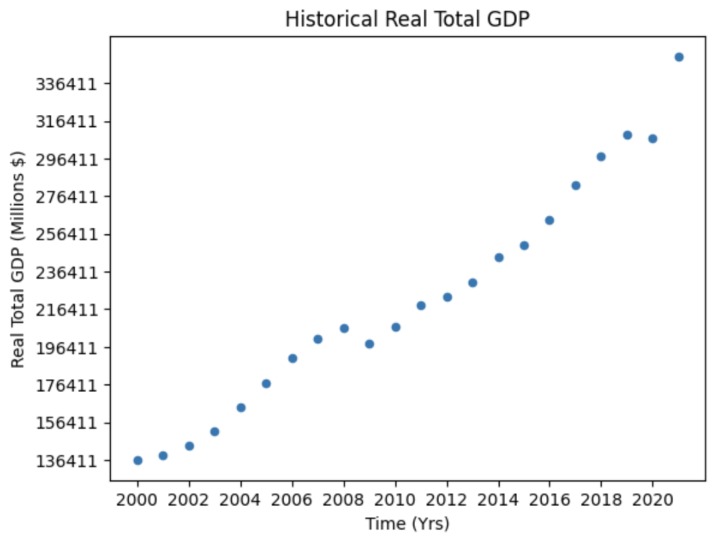



Figure 4: Historical Real Total GDP

# 4 Method

let $x_i$ be an expense category

$x_1$ is health
$x_2$ is education
$x_3$ is social services
$x_4$ is natural resources and economic development
$x_5$ is transportation
$x_6$ is general government expenses
$x_7$ is other

### 4.1 Problem Formulation

The linear programming (LP) problem that we will solve is the following:

minimize $z = \frac{x_1}{a_1} + \frac{x_2}{a_2} + \frac{x_3}{a_3} + \frac{x_4}{a_4} + \frac{x_5}{a_5} + \frac{x_6}{a_6} + \frac{x_7}{a_7}$

subject to

$$
\begin{aligned}
&x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq r \\
&lb_1 \leq \tfrac{x_1}{a_1} \leq ub_1 \\
&lb_2 \leq \tfrac{x_2}{a_2} \leq ub_2 \\
&lb_3 \leq \tfrac{x_3}{a_3} \leq ub_3 \\
&lb_4 \leq \tfrac{x_4}{a_4} \leq ub_4 \\
&lb_5 \leq \tfrac{x_5}{a_5} \leq ub_5 \\
&lb_6 \leq \tfrac{x_6}{a_6} \leq ub_6 \\
&lb_7 \leq \tfrac{x_7}{a_7} \leq ub_7
\end{aligned}
$$

Where $a_i$ is last year's GDP for category $i$, $r$ is last year's government income and $lb_i$, $ub_i$ are the lower-bound and upper-bound (respectively) of a 68% (one standard deviation) confidence interval of the expenditure-to-GDP ratio.

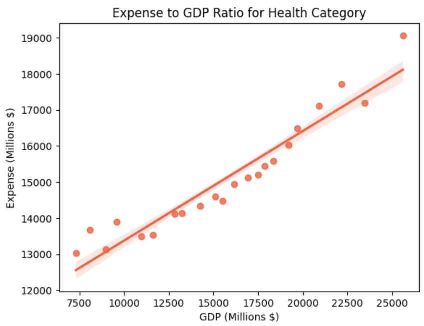
For the constraints of our LP problem, since the total amount we can spend on each expenditure category should be less than or equal to how much the government makes, we will use the government's income from last year.

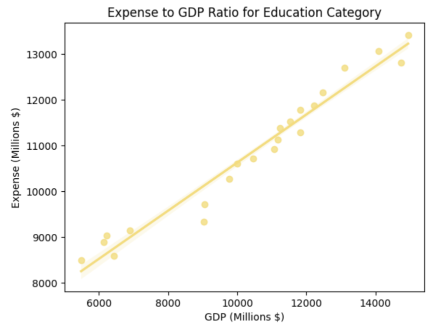
Another set of constraints we need is the historical impact of each category of spending on GDP. For this, we will use the previous years' expenditure-to-GDP ratio to put an upper and lower bound on the range of values for $\frac{x_i}{a_i}$. This also ensures that none of our variables are ever 0 because we will define a lower bound. The use of a confidence interval implies our optimal $x_1 ... x_7$ will be closer to the average of the amount spent in this sector historically than we would if we had chosen to use two or standard deviations from the mean. This means unless certain emergencies break out, such as a pandemic, where the government needs to spend much more in the health sector than they would without a pandemic, our final solution should give an accurate picture and reference for the government's allocation of expenses. This bound can be computed by fitting a line to the historical expenditure vs GDP data(ie, expenditures on the y-axis and GDP on the x-axis.). The slope estimate of this line would be $\frac{expenditure}{GDP}$ and so it would be an estimate of the average expenditure-to-GDP ratio. Then to get a range for our $\frac{x_i}{a_i}$, we would compute a confidence interval for the slope estimate, giving us a lower and upper bound. For example, if for the past 10 years, $\frac{expenditure}{GDP}$ has averaged between 0.2 and 0.4, then we would want to find an $x_i$ where $\frac{x_i}{a_i}$ is minimized and between 0.2 and 0.4. So we would have a constraint: $0.2 \leq \frac{x_i}{a_i} \leq 0.4$. We would do this for all expense categories.

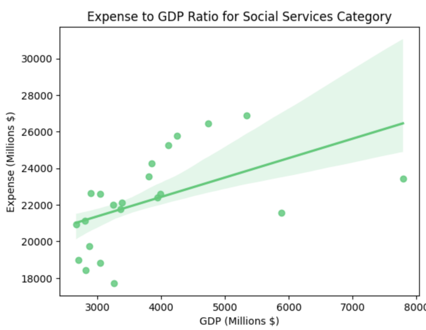
For the linear programming problem we present here, the solution may be degenerate, or we may arrive at a final dictionary that tells us to keep certain expenses 0 (the basic variables in the final dictionary). In either case, it would mean the solution cannot be adopted. For the former case, since we use python packages to solve the LP problem, the library will automatically present one of the optimal solutions and may inform us of the details of that degeneracy, such as the number of degenerate basic feasible solutions. Therefore, assuming the solution we present does not have the latter case problem, even if degenerate, will be optimal and applicable.

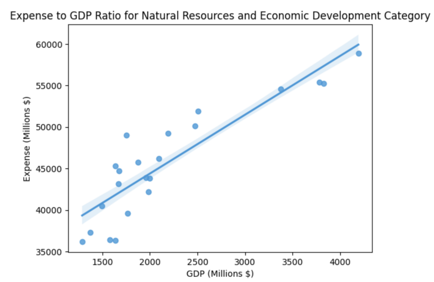
### 4.2 Modeling Historical Data

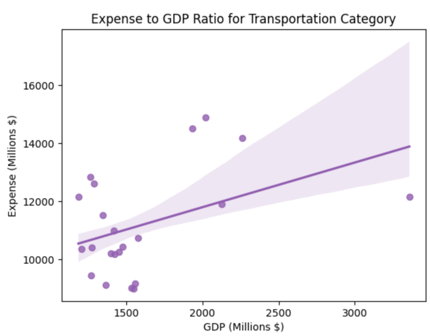
To compute $lb_i$, $ub_i$, we will need to compute the lower and upper bound on the slope estimate of our past expenditure to GDP data. Below are the regression models for each category. The highlighted area is the 68% confidence interval.

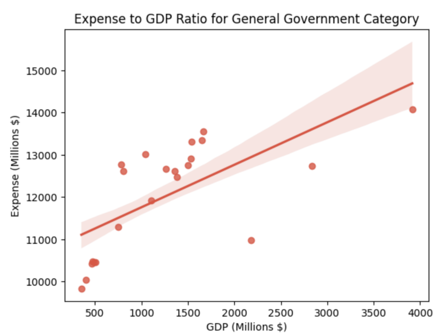

Expense to GDP Ratio for Health Category
Expense (Millions $)
GDP (Millions $)
19000 18000 17000 16000 15000 14000 13000 12000
7500 10000 12500 15000 17500 20000 22500 25000


Expense to GDP Ratio for Education Category
Expense (Millions $)
GDP (Millions $)
13000 12000 11000 10000 9000 8000
6000 8000 10000 12000 14000


Expense to GDP Ratio for Social Services Category
Expense (Millions $)
GDP (Millions $)
30000 28000 26000 24000 22000 20000 18000
3000 4000 5000 6000 7000


Expense to GDP Ratio for Natural Resources and Economic Development Category
Expense (Millions $)
GDP (Millions $)
60000 55000 50000 45000 40000 35000
1500 2000 2500 3000 3500 4000


Expense to GDP Ratio for Transportation Category
Expense (Millions $)
GDP (Millions $)
16000 14000 12000 10000
1500 2000 2500 3000


Expense to GDP Ratio for General Government Category
Expense (Millions $)
GDP (Millions $)
15000 14000 13000 12000 11000 10000
500 1000 1500 2000 2500 3000 3500 4000

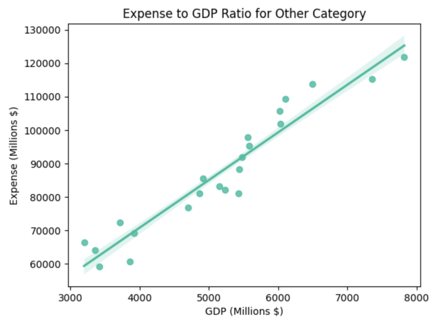



Figure 8: Linear Models of Past Expense to GDP Ratios

### 4.3 Running the LP Problem

Using the data collected by our data analysis, the final LP problem that we will run is the following:

minimize $z = \frac{1}{25605.0}x_1 + \frac{1}{14943.0}x_2 + \frac{1}{7789.0}x_3 + \frac{1}{4191.0}x_4 + \frac{1}{3360.0}x_5 + \frac{1}{3915.0}x_6 + \frac{1}{7821.0}x_7$

subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 62156.0$$
$$0.282779 \leq \frac{1}{25605.0}x_1 \leq 0.324179$$
$$0.501481 \leq \frac{1}{14943.0}x_2 \leq 0.550397$$
$$0.659931 \leq \frac{1}{7789.0}x_3 \leq 1.462201$$
$$6.262428 \leq \frac{1}{4191.0}x_4 \leq 7.911232$$
$$0.790536 \leq \frac{1}{3360.0}x_5 \leq 2.280573$$
$$0.754242 \leq \frac{1}{3915.0}x_6 \leq 1.256824$$
$$13.308526 \leq \frac{1}{7821.0}x_7 \leq 15.223847$$

## 5 Results

After running our LP problem, the results are (rounded to 2 decimal places and in millions of dollars):

$z$ : 22.56

$x_1$ : 7240.56 (Health)

$x_2$ : 7493.63 (Education)

$x_3$ : 5140.20 (Social Services)
$x_4$ : 26245.83 (Natural Resources and Economic Development)
$x_5$ : 2656.20 (Transportation)
$x_6$ : 2952.86 (General Government)
$x_7$ : 104086.00 (Other)

## 6 Discussion

This model is built to maximize GDP, meaning though it produces the optimal allocation of budget to maximize GDP, it may not always predict realistic investments that should actually be put into each sector. For instance, according to our model, the amount spent in the health sector in 2023 is at a historical low (around 7240 million dollars). If we compare this number to the amount spent in 2022 in the health sector (around 27600 million dollars), it is about 3.8 times smaller. Another similar situation occurs in the education sector. Since our model does not take into account the fact that although educational services do not create high GDP instantly, they can have tremendous value in the future. By neglecting this fact, the result from our model ($x_2$) is also 2 times lower than in 2022. However, aside from the unrealistic numbers for some categories, this model gives us, it does give us valuable information. For instance, the healthcare sector costs a lot of money, but the GDP from it is quite small. This suggests that we should look into ways to make the health sector run more efficiently so it costs less money.

Future research should consider more sector-specific data such as industry productivity, labour market conditions, and regional economic factors. Additionally, our model could be extended to include other economic indicators such as employment rates and environmental sustainability. For instance, using the educational services sector example from above, we can build a model that takes into account the future impact of investing in education.

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